

Time Dynamics of Random Access in Cognitive Radio Networks

Tsang-Kai Chang, Kwang-Cheng Chen, and Lizhong Zheng

Abstract—Random access has been widely studied in literature, but its time dynamics remains a pretty open research problem at this time, particularly for cognitive radio networks that are operating most in transient status but being investigated usually in steady-state. Modifying prey-predator model, we therefore consider radio resources as preys and users as predators to dynamically understand the network system behavior. We start from exploring ALOHA, then include the sensing mechanism into the scenario. Furthermore, we incorporate partially or randomly connected graph to practically represent realistic interactions among users and resources. By modeling sensing errors and delay, for the first time, the time dynamics of a cognitive radio network can be fully characterized, and consequently random access operating conditions can be practically understood and specified for network engineering design.

I. INTRODUCTION

Random access protocols define the way for several distributive devices to access the shared spectrum resource. To successfully access the spectrum resource, devices typically sense channels, and then access the channel to transmit the packets based on the sensing result. More delicate random access schemes arise in cognitive radio networks (CRNs) [1], [2]. Due to the fact of overly crowded spectrum, CRNs are aimed to mitigate the spectrum scarcity dilemma by incorporating users with different access priority. In CRNs, the secondary users (SUs) are allowed to use the unoccupied spectrum holes left by the primary users (PUs) opportunistically, as long as the interference to PUs is kept under certain constraint. By allowing heterogenous users to share/compete the radio resource, the sensing and access rules of SUs in CRNs should be carefully designed.

However, the sensing range and access range of any device are limited. The topology of the network plays an important role in random access. One device shall not be able to sense the activities of all other devices, or the sensing may have erroneous result due to fading and interference. In this case, the SUs may intrude the transmission of legitimate PUs, which violates the operation requirement of CRNs. Furthermore, PUs and SUs are able to transmit packets concurrently, as long as they are far apart enough. All the sensing and access relationship among devices can be depicted by the associated graphical topology. Therefore, the network topology should be carefully modeled and analyzed in random access, which is usually ignored in literatures.

This research is supported by the National Science Council, National Taiwan University, and INTEL Corp., under the contract NSC 101-2911-I-002-001 and NTU 102R7501.

In recent decade, great efforts have been done on the sensing and access strategy of CRNs. In [3], [4], the transmission parameters in PHY layer are optimized while limiting the impact on the primary system. However, the system is assumed static with the focus on PHY layer, regardless of the communication behavior of the PUs and the SUs. Other approaches from MAC layer are able to tackle the time dynamics of the system [5]–[9]. A framework of sensing and access in presence of sensing error is proposed in [5], but the interaction among SUs is not addressed. In [6], the medium access protocol allowing concurrent transmission is proposed considering the rate-distance nature in wireless communication, incorporating network topology into protocol design. Timing misalignment between the primary and the secondary system is analyzed, and an implementable asynchronous spectrum access scheme is statistically optimized [7].

Furthermore, the spectrum access is essentially dynamical. Though often more analytically tractable, equilibrium properties do not capture some important behaviors in transient stage. First of all, the availability of channel and the demand of devices vary with time, while most of the time the sensing and access would not execute in the same time. In this case, the devices access the current channel relying on the lagged result of sensing, which leads to the problem of sensing delay. Therefore, a new approach of network dynamics is needed to facilitate dynamic resource allocation in any (cognitive) radio network and to optimize the performance. Recently, an ecological methodology is suggested to analyze time dynamics of CRNs [10].

In this paper, an innovative framework for random access incorporating network topology and time dynamics will be developed. For the topology, graph is a classical tool. As for the dynamics, we borrow the predator-prey model in ecology to manipulate the interaction between users and channel resources. Systems with homogeneous devices, for example slotted ALOHA, will be investigated as a benchmark. This research is then extended to systems with heterogeneous users with distinct access priority. In this work, the imperfections in resource access, including sensing delay, sensing error and network topology, will be more practically considered to supply insights on network design.

The paper is organized as follows: the general framework including topology and dynamics is presented in the following section. In section III, we apply the framework on random access system with homogeneous users, and consider the sensing delay. In section IV, CRNs are considered, as well as the accompanying sensing imperfection. The analytic results are verified by simulations in section V, before the conclusion

is addressed in section VI.

II. TIME DYNAMICS OF RESOURCE ACCESS OVER GRAPH

A. Graph Model

Since a graph is a useful tool to describe the relationship among entities, we use graph to model the topology between users and resources. A graph $G = (V, E)$ is a set of nodes V and edges E , where the node V is model to incorporate two kinds of entities.

- *Resource*
- *Users/Nodes*

Considering two classes of nodes, the edge E in graph stands for three types of relationship between resources and users.

- *Resource link* connects two resources, which describes the topology of the resource availability.
- *Access link* connects one resource and one user, which describes the topological availability of resources to the user.
- *Sensing link* connects two users, and depicts that one user can sense the activity of other users.

To construct the graph model in wireless communications, we should identify the users and resource first. The users access the channels for transmission opportunities. Depending on the network systems, users may be either homogeneous or heterogeneous. For a system with k independent channels, without loss of generality we can view k channels as k radio resource blocks. Since there is no significant structure between resources, we can draw those k channels as an unity, and put emphasis on sensing link and access link in this research.

B. Dynamic Model

In addition to topology, the resource access is essentially dynamic. The demand of users and the availability of resource all change as time evolves. The interplay between resource and users further complicates the dynamics of the system. Therefore, dynamics is critical to the resource access and consuming. In this paper, we consider the topology to be fixed throughout the resource access process, and focus on the dynamics of the demand of users and the availability of resources.

For random access scheme, the number (*i.e.* population) of active users can be benefited by the loss of the population of the radio resources. In this sense, we can find the analogy between predators and users, and that between preys and resources. Classic predator-prey model describes the relation between population densities of prey x_1 and that of predator x_2 [11], which gives

$$\begin{aligned} \dot{x}_1 &= x_1(\alpha - \beta x_2), \\ \dot{x}_2 &= x_2(-\gamma + \delta x_1), \end{aligned}$$

where the parameters α and γ are the birth rate of the prey population and the death rate of the predator population respectively, and the parameters β and δ reflect the predation between two species. With appropriate modification, the dynamics between active users and radio resource can be depicted by the classics predator-prey model through the trace of time [10].

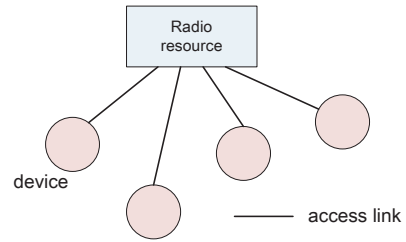


Fig. 1: The resource access graph of ALOHA system. The resource block stands for the k channels conceptually.

III. SYSTEM WITH HOMOGENEOUS USERS

A. Slotted ALOHA

Devices of slotted ALOHA protocol do not sense the channels for transmission. In this sense, slotted ALOHA in medium access control is the simplest case among all resource access problems. Therefore, slotted ALOHA protocol can be served as a benchmark.

1) *System Model*: Consider there are m devices access the base station via k channels by slotted Aloha. In every slot, each device is in one of three states, empty, active, and backlogged. Devices in empty state will receive a new packet with probability λ , and transmit it in the next slot. To transmit, devices choose one of the k channels uniformly, and those transmitting devices are classified as active state. When more than one devices access the same channel, the collision occurs and the involved devices enter backlogged state. Devices in backlogged state will retransmit with probability r until success. We assume that the feedback signal is immediate.

The resource access graph of the ALOHA system is as Fig. 1. The access links of ALOHA system form a star topology. Since the devices in ALOHA do not sense, the sensing link does not exist in the graph.

2) *Dynamic Model*: We define $E(t)$, $A(t)$ and $B(t)$ be the number of users in empty, active, and backlogged state respectively. By the modified predator-prey model, we have the evolving equations of the ALOHA system.

$$\begin{aligned} A(t+1) &= \lambda E(t) + rB(t), \\ B(t+1) &= (1-r)B(t) + \left[1 - \left(1 - \frac{1}{k}\right)^{A(t)-1}\right] A(t). \end{aligned} \quad (1)$$

We consider throughput as the performance measure of the random access system. Throughput is defined as the number of successful transmitted packets per slot normalized to the channel capacity. Based on the proposed model, we have

$$\eta(t) = \frac{1}{k} \left(1 - \frac{1}{k}\right)^{A(t)-1} A(t). \quad (2)$$

B. Resource Availability and Sensing

Opposed to the previous case that channel remains available to users all the time, we assume a predetermined random process to model the resource availability over time. The critical point with varying channel condition is that users

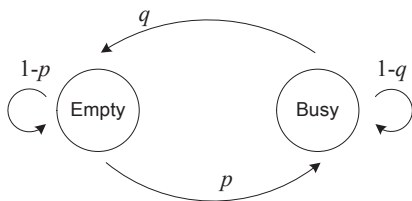


Fig. 2: Markov channel model.

should perform sensing before accessing the channels. Sensing indicates the availability of the channel resource, instead of the activity of other devices, since they are perfectly synchronized by assumption. Note that this is the common CRN model for analyzing SU decision.

In conventional network architecture view, sensing lies in PHY layer while access lies in MAC layer. Considering the interaction between two layers, the layering structure is unnatural in the general resource access view. Therefore, we take the layerless architecture view without presupposed notions of protocol layering [12], in order to emphasize the sensing and access functions in devices.

Let $C_i(t)$ be the state of channel i at slot t , where $C_i(t) \in \{0 \text{ (empty)}, 1 \text{ (busy)}\}$. One of the common channel model is Markov channel model. That is, $\{C_i(t), t = 1, 2, \dots\}$ is a Markov chain as in Fig. 2. The transition probability is defined by $p = \Pr\{C_i(t+1) = 1 | C_i(t) = 0\}$ and $q = \Pr\{C_i(t+1) = 0 | C_i(t) = 1\}$. We assume that all channel i are independent but identical Markov model.

C. Sensing Delay

When the resource availability varies with time, and sensing and access are not performed at the same time, sensing delay is inevitable. That is, users may access the current channel based on the previous sensing result, especially in the case where any channel condition in wireless communication varies rapidly.

To investigate sensing delay, we normalize the slot length to one sensing delay. That is, users with packet should spend one slot to sensing the channel condition before transmission, which we call sensing state. Users will access the channel in the next slot if the channel is currently sensed empty. Active users will access the channel successfully if the channel remains empty and there is no other users access the same channel, while those unsuccessful active users enter backlogged state in the next slot. Empty users and backlogged users enter sensing state with probability λ and r , respectively.

1) *Dynamic Model*: Let $S(t)$ be the number of users in sensing state at slot t . The evolving equations are given by

$$\begin{aligned} E(t+1) &= (1-\lambda)E(t) + p \left(1 - \frac{1}{p\hat{k}(t-1)}\right)^{pA(t)-1} A(t), \\ S(t+1) &= \lambda E(t) + rB(t), \\ A(t+1) &= \frac{\hat{k}(t)}{k} S(t), \end{aligned} \quad (3)$$

where $\hat{k}(t)$ is the number of empty channels at slot t . For the throughput, the exact expression should deal with the collision in the system and the transmission in unavailable channel. We ignore the collision in the unavailable channels, and give the

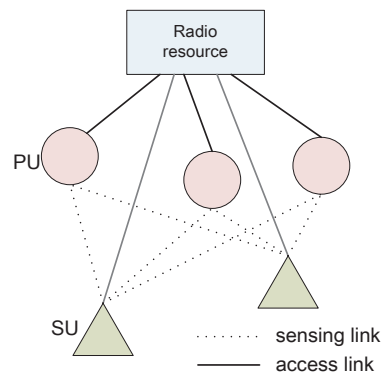


Fig. 3: The resource access graph of the CRN.

throughput in (4). The approximation is valid when the number of channels is relatively large comparing to the number of users, and when the transition probability p is high.

$$\eta(t) = \frac{1}{k} \left(1 - \frac{1}{p\hat{k}(t-1)}\right)^{pA(t)-1} pA(t). \quad (4)$$

2) *Operation Criterion*: Besides the throughput, we consider another performance measure of the impact on unavailable channels. Since users are not supposed to access the unavailable channels, we aim to determine the operation criterion that the influence of unavailable channels is kept under expected value.

For simplicity, we consider the case when $\lambda = r$, and $t \rightarrow \infty$. The stationary probability of available state $\pi_A = q/(p+q)$. The number of users in sensing state in time limit is

$$S = \frac{\lambda}{1 + \lambda + \lambda\pi_A} m. \quad (5)$$

Given a channel is not available, we consider the probability to be accessed by users. Those intruding users sense the available channel in the previous slot. But due to sensing delay, the channel changes the states of availability, resulting in operation mistakes.

$$\begin{aligned} \Pr\{\text{at least 1 users access } C_i | C_i = 1\} \\ = \left[1 - \left(\frac{k-1}{k}\right)^S\right] p. \end{aligned} \quad (6)$$

Proposition 1. *If we require that the probability that the unavailable channel is not accessed is larger than predetermined value β_{delay} , the number of sensing users in time limit should satisfy*

$$S > \frac{\log(1 - \frac{\beta_{delay}}{p})}{\log(\frac{k-1}{k})}. \quad (7)$$

We can observe that the terms in right-hand side in (7) are given by the channel condition and the required value, which can not be modified by the system designer. The design factors, number of users m and the arrival probability λ , determine the number of sensing users in (5), which are in the left-hand side of (7).

IV. COGNITIVE RADIO NETWORKS

A. CRNs with Perfect Sensing

In CRN, users with different priority access the channel concurrently. The SUs have to sense the channel before transmission in order not to interfere with PUs. Firstly, we consider the case that SUs can perfectly avoid the transmission of PUs by sensing. In order to perfectly avoid interfering with PUs, the sensing of SUs must satisfy two criterions. First, each SU is able to sense all PUs, which means that the sensing topology of PUs and SUs are fully-connected. Second, the sensing of SUs has no error. Although these conditions seem strong, the perfect system may serve as an ideal case in studying CRN.

1) *System Model*: The traffic of PUs is not the main concern in this section. We simply consider that the primary system has m_p devices operating in slotted ALOHA as in Section III, with arrival probability λ_p and retransmission probability r_p .

For the secondary system, there are m_s SUs. We assume that the secondary system is synchronized with the primary system. Basically, SUs access the channel by slotted ALOHA with arrival probability λ_s and retransmission probability r_s . However, an active SU chooses one of the k channels uniformly and senses before transmission. We assume that the SUs can sense the activity of PUs at the beginning of the slot. If the channel is already occupied by the PUs, the SU enters backlogged state without transmitting. Otherwise, the SU transmits the packet. We assume that SUs have the immediate feedback of transmission also.

The graph model is as Fig. 3. As discussed previously, there are sensing links connect each SU to all other PUs, while all users have access links to radio resource. The sensing link does not exist between SUs since they are perfectly or pretty much synchronized, which implies that the collision may still occur among SUs to induce the need of MAC [13].

2) *Dynamic Model*: The dynamic model of PUs is given in (1). As for the SUs, since some channels are taken by PUs, the behavior of SUs depends on the number of remaining idle channels $\hat{k}(t)$, which is affected by PUs. The number of attempting SUs is $\lambda_s E_s(t) + r_s B_s(t)$, and the attempting SUs will not sense the PU-occupied channels with probability $\hat{k}(t)/k$, while those attempting SUs who sense the PU-occupied channels enter backlogged state. The dynamics of secondary system is characterized by

$$\begin{aligned} A_s(t+1) &= \frac{\hat{k}(t)}{k} (\lambda_s E_s(t) + r_s B_s(t)), \\ B_s(t+1) &= (1 - r_s) B_s(t) + \left[1 - \left(1 - \frac{1}{\hat{k}(t)}\right)^{A_s(t)-1} \right] A_s(t) \\ &\quad + \left[1 - \frac{\hat{k}(t)}{k} \right] (\lambda_s E_s(t) + r_s B_s(t)). \end{aligned} \quad (8)$$

The throughput of SUs $\eta_s(t)$ can be approximated by

$$\eta_s(t) = \frac{1}{k} A_s(t) \left[1 - \frac{1}{\hat{k}(t)} \right]^{A_s(t)-1}. \quad (9)$$

B. CRNs with Imperfect Sensing

Practically speaking, the assumption of perfect sensing is not realistic. First of all, the fully-connected sensing links between PUs and SUs are impractical. For one device, the sensing range and the transmission range are limited. Once two devices are far apart, one can not sense the other. On the other hand, the erroneous sensing is inevitable in wireless communication due to noise, fading and interference.

Since sensing range is limited, the sensing topology may not be fully-connected. If there is no sensing link between two devices, they do not have the information of the activity of one another. We assume that the transmission of SU will not affect that of PU without sensing link. That is, the system allows simultaneous transmission if the PU and the SU are far apart.

Regarding the sensing error, the operation requirement of CRN may be violated. If SUs can not sense the activity of PUs, miss-sensing SUs will transmit, which surely causes interference to PUs. Furthermore, the system dynamics of PUs are certainly affected by imperfect sensing of SUs also. Thus, the dynamic effect of imperfect sensing should be analyzed. We further propose the operation criterion on the sensing error to guarantee system performance.

1) *System Model*: Instead of fully-connected sensing topology, we assume that each sensing link exists with constant probability ρ between one PU and one SU [14]. The link existence probability ρ represents the connectivity of sensing topology of the network. If there is no sensing link, SU can not sense the PU, nor it will affect the PU. For the secondary system, we assume that all SUs are in the same collision domain. Considering constant link existence probability ρ , the graph model is similar to Fig. 3 with each sensing link present with probability ρ .

For a given occupied channel, we assume a constant miss detection probability ε for each active SU. In other words, if one SU chooses an occupied channel, with probability ε it will not observe the activity of PUs with sensing link and transmit the packet, which causes interference to the active PUs in the same channel.

2) *Dynamic Model*: Given the system conditions in slot t , we can get the expected system behavior in the next slot by modified predator-prey model. The number of SUs with miss sensing is $M_s(t) = (\lambda_s E_s(t) + r_s B_s(t)) \frac{k - \hat{k}(t)}{k} \varepsilon$, where $\hat{k}(t)$ is the number of empty channels. The total active SUs is given by $A_s(t) = \tilde{A}_s(t) + M_s(t)$ with $\tilde{A}_s(t) = (\lambda_s E_s(t) + r_s B_s(t)) \frac{\hat{k}(t)}{k}$. The throughput of SUs may be tedious since we have to distinguish those SUs collided with PUs and those with other SUs. Therefore, we approximate the throughput of SUs by considering only the well-behaved SUs.

$$\eta_s(t+1) = \frac{1}{k} \tilde{A}_s(t) \left[1 - \frac{1}{\hat{k}(t)} \right]^{\tilde{A}_s(t)-1}. \quad (10)$$

For PUs, the probability that one active PU will not meet miss-sensing SUs is $\pi(t) = \left(\frac{k - \hat{k}(t) - 1}{k - \hat{k}(t)} \right)^{M_s(t)}$. The dynamics

evolving equation of primary system is now

$$\begin{aligned} A_p(t+1) &= \lambda_p E_p(t) + r_p B_p(t), \\ B_p(t+1) &= (1-r_p)B_p(t) + \left[1 - \left(1 - \frac{1}{k}\right)^{A_p(t)-1} \pi(t)\right] A_p(t), \end{aligned} \quad (11)$$

with the throughput of PUs

$$\eta_p(t) = \frac{1}{k} \left(1 - \frac{1}{k}\right)^{A_p(t)-1} \pi(t) A_p(t). \quad (12)$$

3) *Operation Criterion*: The sensing error of SUs may intrude the transmission of PUs. Therefore, the performance of PUs is limited by the sensing correctness of SUs. By taking the slotted ALOHA model as a benchmark, the throughput degradation is by a factor $\pi(t)$ in (12), comparing to (2). Therefore, we can derive the operation criterion of the system given the guaranteed PU throughput.

We assume that $\lambda = r$ in the following discussion, and consider the system when $t \rightarrow \infty$. We have $A_p = m_p \lambda_p$ and $A_s = m_s \lambda_s$. The number of available channels is thus given by $\hat{k} = k \left(\frac{k-1}{k}\right)^{\rho A_p}$.

Let η_p^{ALOHA} be the throughput of slotted ALOHA system with the same parameter, which is given in (2). By defining β be the performance degradation of η_p compared to η_p^{ALOHA} , we have the following proposition as an operation criterion for the CRN.

Proposition 2. *For a given guaranteed performance of PUs, $\eta_p > \beta_s \eta_p^{\text{ALOHA}}$, $0 \leq \beta_s \leq 1$, the sensing error probability of SUs should satisfy*

$$\varepsilon < \frac{\log \beta_s}{m_s \lambda_s \frac{k-\hat{k}}{k} \log \left(\frac{k-\hat{k}-1}{k-\hat{k}}\right)}. \quad (13)$$

The term $m_s \lambda_s$ is the designing factor of the secondary system for a given sensing error probability ε . The rest term is determined by the primary system. The operation criterion is considered from the primary system, while the effect to SUs is less obvious. However, the SU access from sensing error does not contribute to the throughput, the sensing error does not affect the SU throughput much, which can be verified from the simulations.

V. SIMULATIONS

A. Sensing Delay in Slotted ALOHA system

The simulation of sensing delay in homogeneous system is shown in Fig. 4. We take the probability that the unavailable channel is accessed by users as the performance measure. In Fig. 4, the analytic probability is from (6), whose effectiveness is verified by the simulated result. The violation probability of unavailable channels increases with the packet arrival probability λ . In the three channel model settings, the stationary distributions are the same, which indicates that the numbers of available channels are identical in stationary sense. The transition probabilities p and q stand for the correlation of delayed sensing result and the current channel condition. Therefore, the case where $p = q = 0.2$ has higher unavailable channel accessed probability than other two cases.

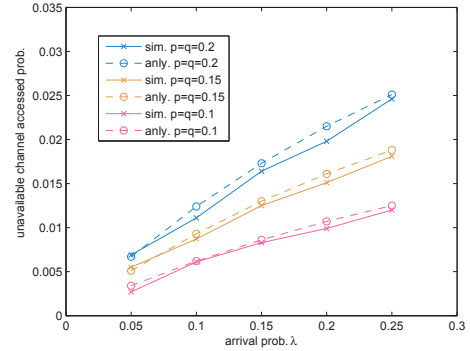


Fig. 4: The collision probability of unavailable channels under various arrival probability. In this simulation, $k = 10$, $m = 7$ and $r = \lambda$. Simulation is $N = 200$ average result with $t = 100$.

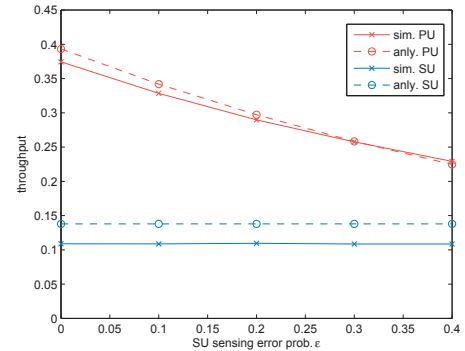


Fig. 5: The system throughput in presence of SU sensing error. In the simulation results, $k = 5$, $\rho = 1$, $m_p = 15$, $m_s = 10$, $\lambda_p = r_p = 0.4$, and $\lambda_s = r_s = 0.6$. Simulation is $N = 200$ average result with $t = 100$.

B. Sensing Error in CRNs

The system performance of CRN with SU sensing error is simulated in Fig. 5, where the analytic results are given by (12) for PUs and by (10) for SUs. The performance of the primary system degrades with the increase of SU sensing error probability ε . However, the SU sensing error probability ε has little effect on the throughput of SUs. Since sensing error of SUs affects the performance of PUs, we further investigate the operation criterion concerning SU sensing error. To guarantee the performance of PUs in presence of SU sensing error, the required sensing error probability is derived in (13). Fig. 6 is the numerical result of the required sensing error probability with various performance degradation. From the numerical result. In the case of $k = 15$, sensing error is not the main concern of the system, and the error criterion gives loose bound. In the case of $k = 5$ where channel resource is relatively scarce, the error criterion suggests the sensing accuracy given the expected performance compared to the ALOHA scheme.

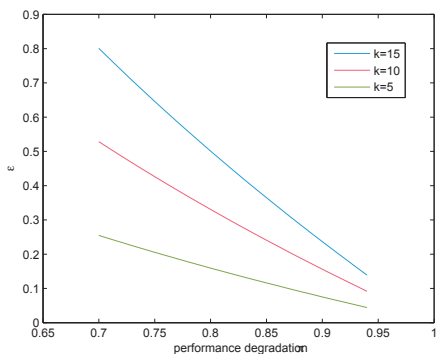


Fig. 6: The required sensing error probability that guarantees a given performance degradation β . The parameters are $\rho = 1$, $m_p = 15$, $m_s = 10$, $\lambda_p = r_p = 0.4$, and $\lambda_s = r_s = 0.6$.

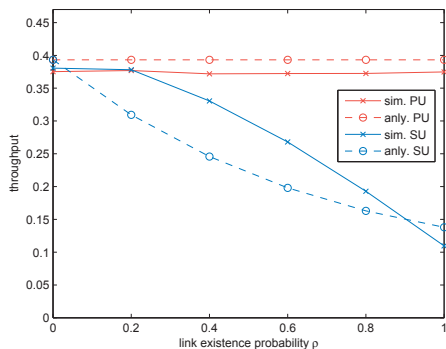


Fig. 7: The system throughput with various link existence probability ρ . In the simulation results, $k = 5$, $\varepsilon = 0$, $m_p = 15$, $m_s = 10$, $\lambda_p = r_p = 0.4$, and $\lambda_s = r_s = 0.6$. Simulation is $N = 200$ average result with $t = 100$.

C. Sensing Topology in CRNs

Fig. 7 is the simulation of system throughput with different sensing topology. For a given sensing range, we can characterize the location dependency of the primary and the secondary system by the link existence probability ρ . When the link existence probability ρ is low, the sensing topology connectivity between PUs and SUs is low, which means that the two systems are distant. In this case, the throughput of SUs benefits from concurrent transmission without affecting PUs, but is limited by the collision among SUs. Although the throughput of SU decreases as the link existence probability increases, the throughput of PUs keeps constant. Therefore, we can separate the causes of throughput degradation of PUs and SUs. Topology effect dominates the throughput of SUs, and the SU sensing error decreases the throughput of PUs.

VI. CONCLUSION

We propose a general framework for the random access in communication systems, including CRNs, to tackle the system topology and dynamics at the same time. The framework can deal with several sensing and access imperfections in an unified way, including sensing error, sensing topology

and sensing delay in this paper. Analytic throughput and system dynamics are derived. With this framework, operation criterions are practically suggested with the presence of those imperfections.

REFERENCES

- [1] J. Mitola and J. Maguire, G.Q., "Cognitive radio: making software radios more personal," *Personal Communications, IEEE*, vol. 6, no. 4, pp. 13–18, 1999.
- [2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 2, pp. 201–220, 2005.
- [3] D. I. Kim, L. B. Le, and E. Hossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 12, pp. 5517–5527, 2008.
- [4] Y. Tachwali, B. F. Lo, I. F. Akyildiz, and R. Agust, "Multiuser resource allocation optimization using bandwidth-power product in cognitive radio networks," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 3, pp. 451–463, 2013.
- [5] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 3, pp. 589–600, 2007.
- [6] S.-Y. Lien, C.-C. Tseng, and K.-C. Chen, "Carrier sensing based multiple access protocols for cognitive radio networks," in *Communications, 2008. ICC '08. IEEE International Conference on*, 2008, pp. 3208–3214.
- [7] Y.-Y. Lin and K.-C. Chen, "Asynchronous dynamic spectrum access," *Vehicular Technology, IEEE Transactions on*, vol. 61, no. 1, pp. 222–236, 2012.
- [8] C. Jiang, Y. Chen, K. J. R. Liu, and Y. Ren, "Renewal-theoretical dynamic spectrum access in cognitive radio network with unknown primary behavior," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 3, pp. 406–416, 2013.
- [9] L. Lai, H. El-Gamal, H. Jiang, and H. Poor, "Cognitive medium access: Exploration, exploitation, and competition," *Mobile Computing, IEEE Transactions on*, vol. 10, no. 2, pp. 239–253, 2011.
- [10] D. Liao, K.-C. Chen, and S.-M. Cheng, "A predator-prey model for dynamics of cognitive radios," *Communications Letters, IEEE*, vol. 17, no. 3, pp. 467–470, 2013.
- [11] J. D. Murray, *Mathematical Biology*. Springer-Verlag, 1989.
- [12] A. Goldsmith, M. Effros, R. Koetter, M. Medard, A. Ozdaglar, and L. Zheng, "Beyond Shannon: the quest for fundamental performance limits of wireless ad hoc networks," *Communications Magazine, IEEE*, vol. 49, no. 5, pp. 195–205, 2011.
- [13] C. Cormio and K. R. Chowdhury, "A survey on MAC protocols for cognitive radio networks," *Ad Hoc Networks*, vol. 7, no. 7, pp. 1315–1329, 2009.
- [14] K.-C. Chen, "Medium access control of wireless LANs for mobile computing," *Network, IEEE*, vol. 8, no. 5, pp. 50–63, 1994.